Closing tonight (11pm): 10.1
Closing Wed: 2.1
Closing Fri: 2.2
Closing next Mon (no class): 2.3
Warning: Big assignments, see hints in newsletter and use the MSC!

*Entry Task*:
Draw quick rough graphs of

1. $f_1(x) = \ln(x)$
2. $f_2(x) = \sin(x)$
3. $f_3(x) = |x| + 1$
4. $f_4(x) = \tan^{-1}(x)$
5. $f_5(x) = \frac{1}{x^2}$
6. $g(x) = \frac{x^2-4}{x-2}$
7. $h(x) = \begin{cases} x^2 & \text{, if } x \neq 0; \\ 3 & \text{, if } x = 0. \end{cases}$

## 2.2 Limits

When we write

$$\lim_{x \to a} f(x) = L$$

we say "the **limit** of f(x), as x approaches a, is L".

and we mean

as *x* takes on values closer and closer to *a*,

*y = f(x)* takes on values closer and closer to L.

This notation gives us a way to
discuss what is happen "near" a value
*x = a* (but not at the value).

## ***One-sided limits***

When we write

$$\lim_{x \to a^-} f(x) = L$$

we say “the limit of f(x), as x approaches a **from the left**, is L”.
and we mean

as *x* takes on values closer to and **from the left** of *a*,
*y = f(x)* takes on values closer and closer to L.

Similarly,

$$\lim_{x \to a^+} f(x) = L$$

we say “the limit of f(x), as x approaches a **from the right**, is L”.

Note:

$$\lim_{x \to a} f(x) = L \text{ if and only if both } \begin{cases} \lim_{x \to a^-} f(x) = L \\ \lim_{x \to a^+} f(x) = L \end{cases}$$

## 2.3 Limit Laws and Strategies

Some Basic Limit Laws:

1. $\lim_{x \to a} c = c$

2. $\lim_{x \to a} x = a$

3. $\lim_{x \to a} [f(x) + g(x)]$
$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

4. $\lim_{x \to a} [f(x) g(x)]$
$= \lim_{x \to a} f(x) \lim_{x \to a} g(x)$

5. If $\boldsymbol{\lim_{x \to a} g(x) \neq 0}$, then
$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

Examples:

1. $\lim_{x \to -7} 10 = 10$

2. $\lim_{x \to 14} x = 14$

3. $\lim_{x \to -2} [x + 6] = \lim_{x \to -2} x + \lim_{x \to -2} 6$

4. $\lim_{x \to 5} [2x^2] = \lim_{x \to 5} 2 \lim_{x \to 5} x \lim_{x \to 5} x$

5. $\lim_{x \to 4} \left[\frac{x + 2}{x^2}\right] = \frac{\lim_{x \to 4} (x + 2)}{\lim_{x \to 4} x^2}$

*Limit Flow Chart for*

$$\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right]$$

1. Try plugging in the value.
   **If denominator ≠ 0, done!**

2. **If denom = 0 & numerator ≠ 0**, the answer is -∞, +∞ or DNE. Examine the sign (pos/neg) of the output from each side.

3. **If denom = 0 & numerator = 0**, Use algebraic methods to simplify and cancel until one of them is not zero.

**For the den = 0, num = 0 case**, here is a summary of some algebra to try:

*Strategy 1*: Factor/Cancel
*Strategy 2*: Simplify Fractions
*Strategy 3*: Expand/Simplify
*Strategy 4*: Multiply by Conjugate
*Strategy 5*: Change Variable
*Strategy 6*: Compare to other functions (Squeeze Thm)